| **Date: 9/12/23** | **FIRST MODEL EXAMINATION-2024** | **Max marks: 80** |
| **GRADE: XII** | **MATHEMATICS (041)** | **Time: 3 Hours** |

General Instructions:

1.This question paper consists of **five sections A, B, C, D and E**. Each section is compulsory.

2.**Section A** comprises of 18 MCQs of one mark each(from Q01-Q18) and Assertion-Reasoning based questions (from Q 19- Q20)

3.**Section B** comprises of 05 Very Short Answer(VSA)-type questions of 2 marks each(from Q21-Q25).

4.**Section C** comprises of 06 Short Answer (SA)-type questions of 3 marks each (from Q26-Q31) .

5.**Section D** comprises of 04 Long Answer (LA)-type questions of 5 marks each (from Q32-Q35)

6.**Section E** comprises of 03 Case-study based questions of 4 marks each (fromQ36- Q38)

7. There is no overall choice. However, internal choice has been provided in some of the questions. You must attempt only one of the alternatives in all such questions.

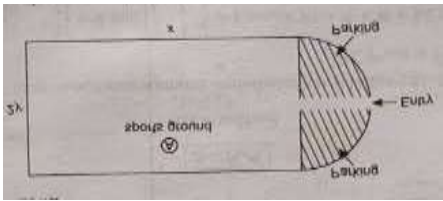|	SECTION A	
1	If $A = \begin{bmatrix} 1 & -2 & 4 \\ 2 & -1 & 3 \\ 4 & 2 & 0 \end{bmatrix}$ is the adjoint of a square matrix B, then B^{-1} is equal to (a) $\pm A$ (b) $\pm\sqrt{2}A$ (c) $\pm\dfrac{1}{\sqrt{2}}B$ (d) $\pm\dfrac{1}{\sqrt{2}}A$	
2	Let $A = \{1, 2, 3, ..., 100\}$. Let a relation R be defined on A, given by $R = \{(x, y) : xy \text{ is a perfect square}\}$.Then the equivalence class [2] is (a) {2, 8, 18, 32, 50} (b) {2, 8, 18, 32} (c) {2, 8, 18, 32, 50, 72, 98} (d) None of these	
3	The value of $\tan^2(\sec^{-1}2) + \cot^2(\csc^{-1}3)$ is (a) 5 (b) 11 (c) 13 (d) 15	
4	If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and $A + A' = I$, then the value of α is	

	(a) $\dfrac{\pi}{6}$	(b) $\dfrac{2\pi}{3}$	(c) π	(d) $\dfrac{\pi}{3}$	

5	If If A is a non-singular matrix of order 3 and $|A| = -4$, find $|\text{adj } A|$. (a) 4 (b) 16 (c) 64 (d) $\dfrac{1}{4}$	

6	If A is a square matrix of order 3 such that $|A| \neq 0$, then which of the following is not true? (a) $|\text{adj}.A| = |A|^2$ (b) $|A| = |A'|$ (c) $\left|A\right|^{-1} = \left|A^{-1}\right|$ (d)A is a singular matrix	

7	Find the points where the function f(x) = $[x]$, $0 \leq x < 3$ is not differentiable. a) 1 (b) 1 & 2 (c) 1,2 & 3 (d) 0,1,& 2	

8	If $y = \tan^{-1}(e^{2x})$, then $\dfrac{dy}{dx}$ is equal to (a) $\dfrac{2e^{2x}}{1+e^{4x}}$ (b) $\dfrac{1}{1+e^{4x}}$ (c) $\dfrac{2}{e^{2x}+e^{-2x}}$ (d) $\dfrac{1}{e^{2x}-e^{-2x}}$	

9	The function $f(x) = \begin{cases} x^2 & \text{for } x < 1 \\ 2-x & \text{for } x \geq 1 \end{cases}$ is (a) not differentiable at $x = 1$ (b)differentiable at $x = 1$ (c)not continuous at $x = 1$ (d)neither continuous nor differentiable at $x = 1$	

10	The number of corner points of the feasible region determined by the constraints $x - y \geq 0$, $2y \leq x+2$, $x \geq 0$, $y \geq 0$ is (a) 2 (b) 3 (c) 4 (d) None of these	

11	In $\triangle ABC$, $\overrightarrow{AB} = \hat{i} + \hat{j} + 2k$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4k$. Let D is the mid-point of BC, then vector $\overrightarrow{AD}$ is equal to (a) $4\hat{i} + 6k$ (b) $2\hat{i} - 2\hat{j} + 2k$ (c) $\hat{i} - \hat{j} + k$ (d) $2\hat{i} + 3k$	

12	What is the product of the order and degree of the differential equation $\dfrac{d^2y}{dx^2}\sin y + \left(\dfrac{dy}{dx}\right)^3 \cos y = \sqrt{y}$?	

	(a) 3 (b) 2 (c) 6 (d) not defined	
13	if $\|\vec{a}\| = \frac{\sqrt{3}}{2}, \|\vec{b}\| = 4$ and angle between them is 60^0 then the value of $\vec{a}.\vec{b}$ is equal to (a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $-\sqrt{3}$ (d) none of these	
14	The confidence gained by playing x games of tennis at a trial function is given by $C(x) = 11 + 15x + 6x^2 - x^3$. Then, the marginal confidence gained after playing 5 games, is (a) $15 + 12x - 3x^2$ (b) $15 + 2x - 3x^2$ (c) $15 + 2x + 3x^2$ (d) 0	
15	A vector of magnitude 5 and perpendicular to vectors $\hat{i} - 2\hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} - 3\hat{k}$ is a) $\dfrac{5\sqrt{3}\left(\hat{i} + \hat{j} + \hat{k}\right)}{3}$ b) $\dfrac{5\sqrt{3}\left(\hat{i} - \hat{j} + \hat{k}\right)}{3}$ c) $\dfrac{5\sqrt{3}\left(\hat{i} - \hat{j} - \hat{k}\right)}{3}$ d) $\dfrac{5\sqrt{3}\left(\hat{i} + \hat{j} - \hat{k}\right)}{3}$	
16	If a vector makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and y-axis, then the angle which it makes with positive z-axis is (a) $\dfrac{\pi}{4}$ (b) $\dfrac{3\pi}{4}$ (c) $\dfrac{\pi}{2}$ (d) 0	
17	If the sum of numbers obtained on throwing a pair of dice is 9, then the probability that number obtained on one of the dice is 4, is (a) $\dfrac{1}{9}$ (b) $\dfrac{4}{9}$ (c) $\dfrac{1}{18}$ (d) $\dfrac{1}{2}$	
18	$\displaystyle\int_{0}^{\frac{\pi}{6}} \sec^2\left(x - \frac{\pi}{6}\right) dx$ is equal to (a) $\dfrac{1}{\sqrt{3}}$ (b) $-\dfrac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $-\sqrt{3}$	
	Followings are Assertion-Reason based questions. In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.	

	(a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true and R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.	
19	**Assertion (A):** The range of the function $f(x) = 2\sin^{-1}x + \dfrac{3\pi}{2}$, where $x \in [-1, 1]$, is $\left[\dfrac{\pi}{2}, \dfrac{5\pi}{2}\right]$. **Reason (R):** The range of the principal value branch of $\sin^{-1}(x)$ is $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$	
20	**Assertion (A) :** A line through the points $(4,7,8)$ and $(2,3,4)$ is parallel to a line through the points $(-1,-2,1)$ and $(1,2,5)$. **Reason (R) :** Lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are parallel, if $\vec{b}_1 \cdot \vec{b}_2 = 0$.	
21	a)From the graph given below , write the function, its domain and range OR b)Find the domain of the function y = cos⁻¹(3x − 1)	2
22	Out of a group of 60 architects, 40 are qualified and co-operative, while the remaining are qualified and remains reserved. Two architects are selected from the group at random. Find the Probability distribution and the expected number(mean) of architects who are qualified and co-operative.	2

23	a) Find the derivative of $\tan^{-1} x$ with respect to log (x) where $x \in$ (1 ,∞). OR (b) Differentiate the following function with respect to x : $(\cos x)^{x}$, where$x \in (0, \pi 2)$.	2
24	Find the intervals in which the function f(x) defined by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is strictly increasing or strictly decreasing.	2
25	A police man fires 4 bullet on a dacoit. The probability that the dacoit will be killed by one bullet is 0.6. What is the probability that the dacoit is still alive.	2
	SECTION C	
26	Solve the differential equation $\frac{dy}{dx} = (1 + x^2)(1 + y^2)$	3
27	Evaluate : $\int \frac{x^2+ x+1}{(x+1)(1+ x^2)} \, dx$ OR Evaluate : $\int (\frac{(x^2+ 1)e^x}{(x+1)^2} \, dx$	3
28	Find the particular solution of the D.E. : $\frac{dy}{dx} - 3y \cot x = \sin 2x$, given that $y = 2$ when $x = \frac{\pi}{2}$.	3
29	Find the shortest distance between the lines $\vec{r} = 4\hat{i} - 3\hat{j} + \lambda(\hat{i} + 2\hat{j} - 2k)$ and $\vec{r} = \hat{i} + \hat{j} - 2k - \mu(2\hat{i} + 4\hat{j} - 4k)$.	3
30	Evaluate $\int \frac{2x-3}{x^2+ 3 x-18} \, dx$	3
31	Solve the following linear programming problem graphically. Maximize $z = 5x + 3y$ Subject to the constraints $3x + 5y \leq 15, \; 5x + 2y \leq 10, \; x \geq 0, \; y \geq 0.$	3
	SECTION D	
32	Determine the area of the region bounded by the curves $x^2 = y, y = x + 2$ an axis, using the concept of integration. **OR**	5

	Find the area of the region bounded by the line $y = \sqrt{3}x$, the curve $x^2 + y^2 = 4$ and the x-axis in the first quadrant.	
33	Find the image of the point P(2, -1, 5) in the line $\dfrac{x-11}{10} = \dfrac{y+2}{-4} = \dfrac{z+8}{-11}$. Also, find the length of the perpendicular from the point P(2, -1, 5) to the line. OR Check whether the lines $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-5}{4}$ and $\dfrac{x-4}{5} = \dfrac{y-1}{2} = z$ are skew lines or not. Also find the distance between them.	
34	Evaluate: $\int_{-1}^{3/2} |x\sin(\pi x)|\, dx$ OR $Evaluate \displaystyle\int_{0}^{\frac{\pi}{4}} \dfrac{\sin x + \cos x}{9 + 16\,\sin 2x}\, dx$	5
35	Let R be a relation on the set of real numbers **R** defined by R = {(a,b) ; a,b ϵ **R** and a-b+$\sqrt{3}$ is an irrational number}. Is R reflexive, symmetric and transitive?	5
	SECTION E	

36.	Read the following passage and answer the questions given below:
	
	 During the time of need and otherwise also people help the needy. In a survey it was found that out of 200 people surveyed in a city 50 help the needy on a regular basis, 120 contribute to Prime Minister Relief Fund and the rest help through NGO;s. A person is selected who needs a help, the probabilities of help through persons on regular basis, from Prime Minister relief fund and through NGO's are 0.15, 0.06 and 0.10 respectively.
	
	   a) Find the probability that the needy person received the help..
	   b) Find the probability of helping the needy through Prime Minister relief fund.
	  OR
		4

	Find the probability that needy person was helped through person on regular basis. c) Find the probability of a help through NGO's.	

37	The government of a state, which has mostly hilly area decided to have adventurous playground on the top of the hill having plane area and space for 10000 persons to sit at a time. After survey it was decided to have rectangular playground with a semicircular parking at one end of playground. The total perimeter of the field is measured as 1000m.
	
	Based on the above passage answer the following questions:
	   a) Find the relation between x and y.
	   b) Find the area of the sports ground in terms of x.
	   c) Find the value of x for which sports ground has the maximum area. Also find the maximum area.
	 OR If government wants to maximize the area including the parking space, find the value of y to maximize the total area.
		2
		
		2

38.	The Resident Welfare Association of a colony has 3 different subcommittees with total of 12 members. First subcommittee is adult education committee, which looks after the literacy needs of the adults, the second subcommittee is health and cleanliness committee, which looks after health and cleanliness needs of the colony and third subcommittee is safety committee, which looks after the safety needs of the colony. The number of members of the first subcommittee is half of the sum of the members of the other two subcommittees and the number of members of the second subcommittee is the sum of the members of the other two sub committees.
	   a) Reduce the information in the form of algebraic statements
	   b) find the number of members in each committee using matrices.
	
	 OR Two formers Ramkrishan and Gurucharan Singh cultivate only three varieties of rice namely Basmati, Permal and Naura. The sale (in Rupees) of these varieties of rice by both the farmers
		1
		
		3

in the month of September and October are given by the following matrices A and B.

September Sales (In rupees):

$$\begin{array}{ccc} Basmati & Permal & Naura \\ 10{,}000 & 20{,}000 & 30{,}000 \\ 50{,}000 & 30{,}000 & 10{,}000 \end{array} \quad \begin{array}{l} Ramakrisan \\ Gurucharan\ Singh \end{array}$$

October Sales (In rupees):

$$\begin{array}{ccc} Basmati & Permal & Naura \\ 50{,}000 & 10{,}000 & 6{,}000 \\ 20{,}000 & 10{,}000 & 10{,}000 \end{array} \quad \begin{array}{l} Ramakrisan \\ Gurucharan\ Singh \end{array}$$

On the basis of above information answer the following questions

(i) What is the order of the matrix AxB? — 1

(ii) If a_{ij} is the element of the matrix A and if b_{ij} is the element of matrix B, then what is the result of $a_{23} x b_{22}$? — 1

(iii) Find the combined sale in September and October for each farmer and each variety. — 2